A Simple Semantics for Aristotelian Apodeictic Syllogistics

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AiML 2010
27 August 2010, Moscow

[1]Funded by the NWO project "Dialogical Foundations of Semantics" (DiFoS) in the ESF EuroCoRes programme LogICCC (LogICCC-FP004; DN 231-80-002; CN 2008/08314/GW).

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Let $A, B, C \ldots$ range over a set TERM of terms.

We have four types of assertoric categorical propositions:

AaB	'A belongs to all B'	$\leftrightarrow$ 'All B are A' (universal affirmative)
AeB	'A belongs to no B'	$\leftrightarrow$ 'No B is A' (universal negative)
AiB	'A belongs to some B'	$\leftrightarrow$ 'Some B is A' (particular affirmative)
AoB	'A does not belong to some B'	$\leftrightarrow$ 'Some B is not A' (particular negative)

The term preceding the copula is the predicate term and the term succeeding it is the subject term.

Aristotelian (assertoric) syllogistics (2)

Definition

A triple $\mathcal{S} = \langle M, m, c \rangle$, where M, m, and c are categorical propositions, is a *syllogism* if M, m, and c contain exactly three distinct terms, of which the predicate of c (called the major term) appears in M and the subject of c (called the minor term) appears in m, and M and m share a term (called the middle term) which is not present in c.

We call M the major premise, m the minor, and c the conclusion.

Aristotelian (assertoric) syllogistics (3)

The three ways that major, minor, and middle terms in the premises can be arranged are called figures:

1st	2nd	3rd
A — B	B — A	A — B
B — C	B — C	C — B
A — C	A — C	A — C

Figure: The Three Figures

A figure with three copulae added is called a ‘mood’.

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One of the most difficult to understand logical systems in history. Starting with some of his own students, many have considered Aristotle's modal syllogistic to be anywhere from confused to simply wrong.

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- Theophrastus: both syllogisms are invalid—nothing follows when one premise is necessary and the other assertoric.
- Łukasiewicz: both syllogisms are valid.

Attempts to give a consistent interpretation

Łukasiewicz:

> *Aristotle's modal syllogistic is almost incomprehensible because of its many faults and inconsistencies . . . modern logicians have not as yet been able to construct a universally acceptable system of modal logic which would yield a solid basis for. . . Aristotle's work.*

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Recent semantic approaches: Johnson (1989, 1993), Thomason (1993), and Malink (2006).

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> *The key to Aristotle's theory lies, I am convinced, in viewing the theory of modal syllogisms of the Analytica Priora in the light of the theory of scientific reasoning of the Analytica Posteriora.*

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> *[A] rule that is necessarily (say) applicable to all of a group will be necessarily applicable to any sub-group, pretty much regardless of how this sub-group is constituted. On this view, the necessary properties of a genus must necessarily characterize even a contingently differentiated species. If all elms are necessarily deciduous, and all trees in my yard are elms, then all trees in my yard are necessarily deciduous (even though it is not necessary that the trees in my yard be elms).*

Rescher's interpretation, revised by McCall

This interpretation only works for first-figure syllogisms with mixed necessary and assertoric premises.

In the case of second and third figure syllogisms, such as *XLL* Camestres, the minor premise is the general rule, and the major premise is the special case.

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Revision: The general rule is the premise in which the middle term is distributed, with two exceptions:

1. general rules cannot be particular (*XLL* Baroco)
2. special cases cannot be negative (*XLL* Felapton, *XLL* Bocardo)

General rules and special cases

Definition

A premise in a syllogism $\mathcal{S}$ is a *general rule* if (1) the middle term is distributed and (2) it is not particular.

A premise in a syllogism is a *special case* (1) if the other premise is a general rule and (2) only if it is not negative.

Simple syllogistic models

Definition

A *simple syllogistic model* is a tuple $\mathfrak{M}^{\mathrm{S}} = \langle W, D, R, O, V \rangle$ where W is a set (of possible worlds); D is a set (of objects); $R \subseteq W \times W$ is reflexive, transitive, and symmetric; for $w \in W, O(w) \subseteq D$ is the set of objects existing in w; and for $A \in \mathrm{TERM}, V(A) \subseteq D$ is the set of objects in the extension of a term A.

Truth conditions for assertoric and modal propositions

$V(A)$ is extended naturally to $V'(A,w) = V(A) \cap O(w)$. The truth conditions for the assertoric propositions are as expected:

Definition

$$\begin{array}{lll}
\mathfrak{M}^{\mathrm{S}}, w \vDash AaB & \text{iff} & V'(B,w) \neq \emptyset \text{ and } V'(B,w) \subseteq V'(A,w). \\
\mathfrak{M}^{\mathrm{S}}, w \vDash AiB & \text{iff} & V'(A,w) \cap V'(B,w) \neq \emptyset. \\
\mathfrak{M}^{\mathrm{S}}, w \vDash AeB & \text{iff} & V'(A,w) \cap V'(B,w) = \emptyset. \\
\mathfrak{M}^{\mathrm{S}}, w \vDash AoB & \text{iff} & V'(B,w) = \emptyset \text{ or } V'(A,w) \not\subseteq V'(B,w).
\end{array}$$

Since R is an equivalence relation on W, the modalities L and M are the usual S5 modalities.

Validity of a syllogism

Definition (Conditionalization)

For a model $\mathfrak{M}^{\mathrm{S}}$ and formula φ, the *update of* $\mathfrak{M}^{\mathrm{S}}$ *by* φ is the model $\mathfrak{M}^{\mathrm{S}} \upharpoonright \varphi = \langle W \upharpoonright \varphi, D, R \upharpoonright \varphi, O \upharpoonright \varphi, V \upharpoonright \varphi \rangle$ where $W \upharpoonright \varphi = \{w \in W : \mathfrak{M}^{\mathrm{S}}, w \vDash \varphi\}$; D is unchanged; and $R \upharpoonright \varphi$, $O \upharpoonright \varphi$, and $V \upharpoonright \varphi$ are the restrictions of the original relations and functions to $W \upharpoonright \varphi$.

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Definition (Validity)

A syllogism $\mathcal{S}$ with special case s is valid for any simple model $\mathfrak{M}^{\mathrm{S}}$ and $w \in W$ iff (i) $\mathfrak{M}^{\mathrm{S}}, w \vDash M$ and (ii) $\mathfrak{M}^{\mathrm{S}}, w \vDash m$ imply (iii) $\mathfrak{M}^{\mathrm{S}} \upharpoonright s, w \vDash c$.

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Note:
A syllogism $\mathcal{S}$ can be valid at world W in a simple model $\mathfrak{M}^{\mathrm{S}}$ even if the premises are true at w and the conclusion false.

An example (validity)

LXL Barbara $\langle LAaB, BaC, LAaC \rangle$ is valid.

Proof.

Suppose that (i) $\mathfrak{M}^{\mathrm{S}}, w \vDash LAaB$ and (ii) $\mathfrak{M}^{\mathrm{S}}, w \vDash BaC$; we need to show that (iii) $\mathfrak{M}^{\mathrm{S}} \upharpoonright BaC, w \vDash LAaC$. By the definition of model restriction, $\mathfrak{M}^{\mathrm{S}} \upharpoonright BaC, w \vDash LBaC$ since the only worlds remaining are those worlds where BaC is true. From assumption (i), we also have that $\mathfrak{M}^{\mathrm{S}} \upharpoonright BaC, w \vDash LAaB$. □

An example (invalidity)

XLL Barbara $\langle AaB, LBaC, LAaC \rangle$ is not valid:

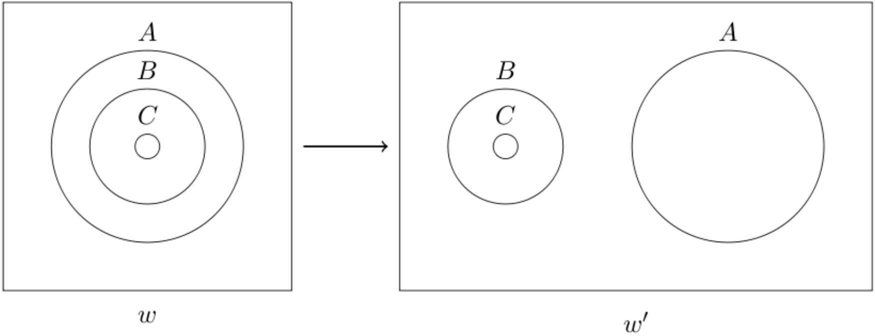

Figure 2. Countermodel for *XLL* Barbara.

Adequacy of the semantics

- All of purely assertoric syllogisms are valid.
- The four perfect *LLL* syllogism are valid.
- Every valid second- or third-figure *LLL* syllogism can be derived from one of the four perfect first-figure assertoric syllogism by means of simple conversion and accidental conversion.
- Simple and accidental conversion are sound.
- The validity of the *XLL* and *LXL* syllogisms corresponds directly to the non-modalization of the premise other than one in which the middle term is distributed.

Limitations of our semantics (1)

This def. does not work for the $L - X - M$ fragment: It validates MXM Barbara and MXM Celarent (which should not be valid).

Proof.

When we update with the special case (the minor premise), $\mathfrak{M}^{\mathrm{S}} \upharpoonright m, w \models m$ for all $w \in W$; then either the world which made the major premise true is still in the model, in which case the conclusion must also be true, or the conclusion is false in the updated model, but we have then falsified the major premise. □

Limitations of our semantics (2)

More generally:

- When the special case is non-modal, the update procedure will always promote an assertoric premise to a necessary one.
- When the special case is modal, since R is an equivalence relation, model reduction by a modal formula does not change the model.

The culprit

Contraposition is *not* sound on these semantics: There is no correlation between the premise that is modal and the premise that distributes the middle term in a syllogism and its contraposed form.

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Example: On the other hand, in *XMM* Camestres, the minor is modalized and contains the distributed middle, whereas in its contraposition, *XLL* Darii, the minor premise remains modalized, but the middle term is distributed in the major premise.

The big question

How can we extend the semantics to the $L - X - M$ fragment?

Conclusions

- We have provided a semantics which validates the axiomatization for the $L - X$ fragments of Aristotle's modal syllogistics.
- These semantics, which crucially rely on a model update process, are much simpler than those found in previous literature.
- They take seriously Rescher's proposal that a modal syllogism should be interpreted as making a claim about a specific case of a general scientific principle.
- This independent motivation for the use of the dynamic upgrade of the premise which is special case means that our system is not *ad hoc*, but instead has good philosophical grounding.
- It is not straightforward to extend this definition to a larger fragment.